
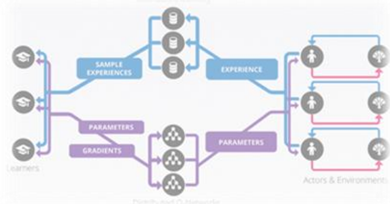

Ant Colony Optimal

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Introduction

ACO

· Ant colony optimization(ACO) is a search algorithm, for solving combinatorial optimization Problems which can be reduced to finding good paths through graph.

· ACO is search algorithm aiming to search for an optimal path in a graph, based on the behavior of ants seeking a path between the nest and food source.

Introduction

Ant foraging behavior

- The ants search randomly around the nest looking for food.
- When the ants find food, they laid pheromone on their path return to the nest.

Introduction

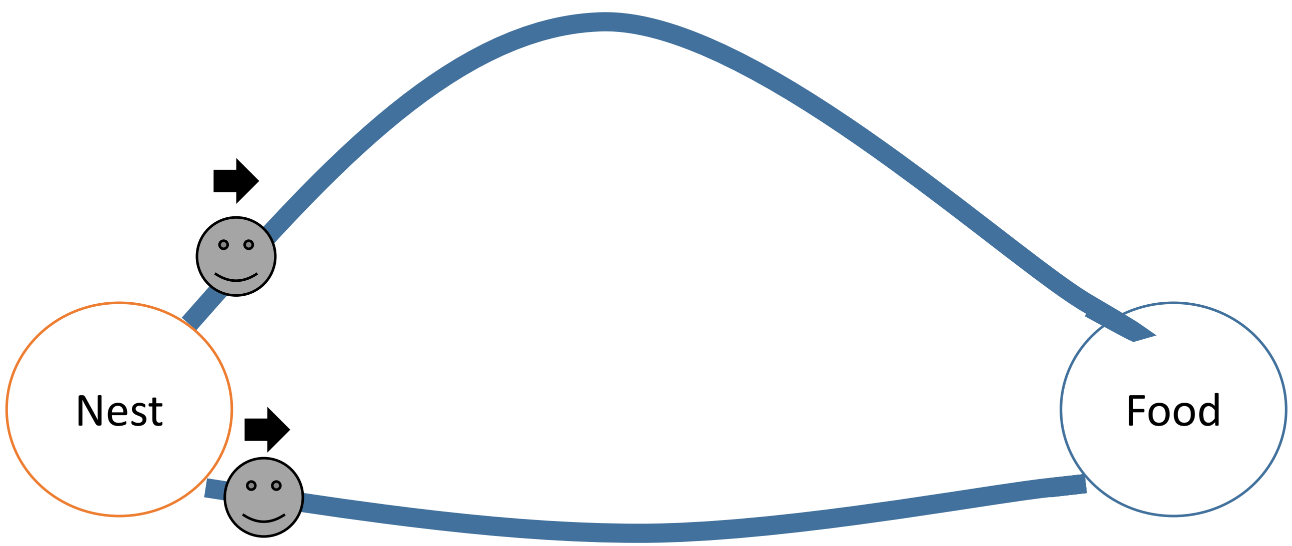
Pheromone

- The pheromone deposited while walking between food sources and the nest.
- It can be used to mark a path on the ground that others can follow.
- Pheromones are so attractive that ants want to follow.
- The Pheromones evaporate gradually over time.

Introduction

Ant foraging behavior

- Ant foraging search by pheromone trails

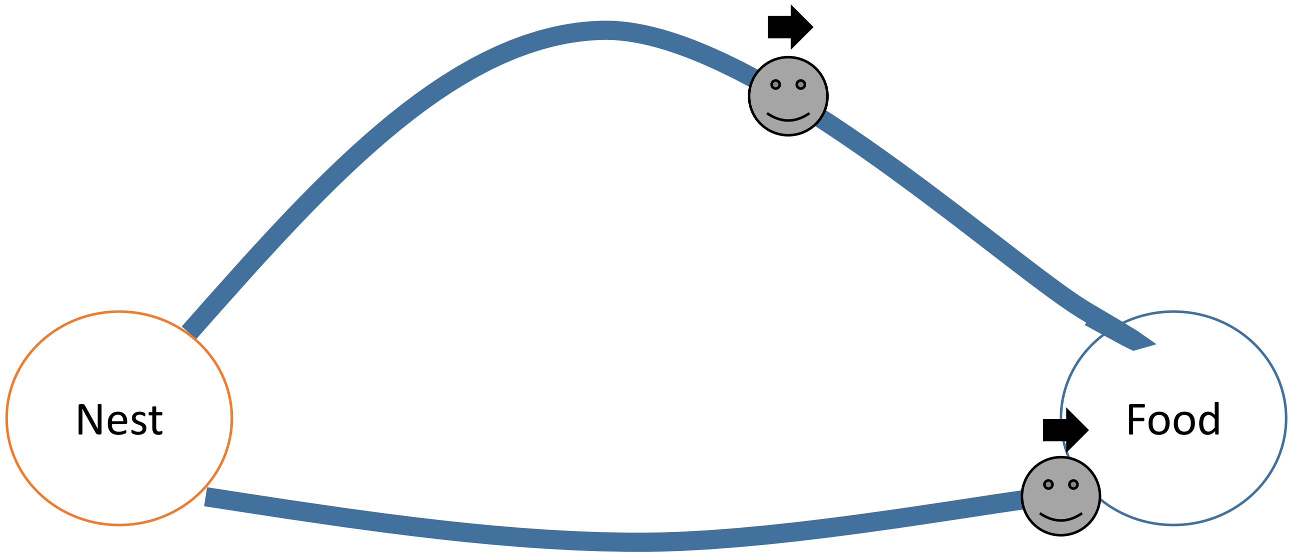


When ants leave their nest to search for a food source,
They randomly rotate around an obstacle

Introduction

Ant foraging behavior

- Ant foraging search by pheromone trails

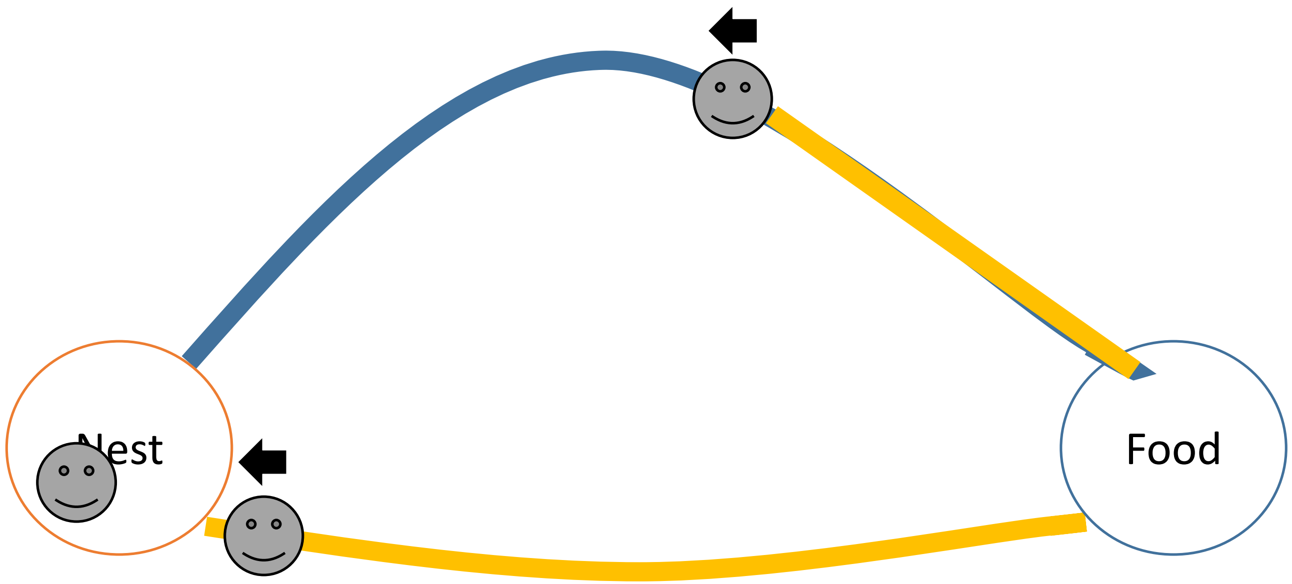


Initially the pheromone deposits will be the same for each path.

Introduction

Ant foraging behavior

- Ant foraging search by pheromone trails

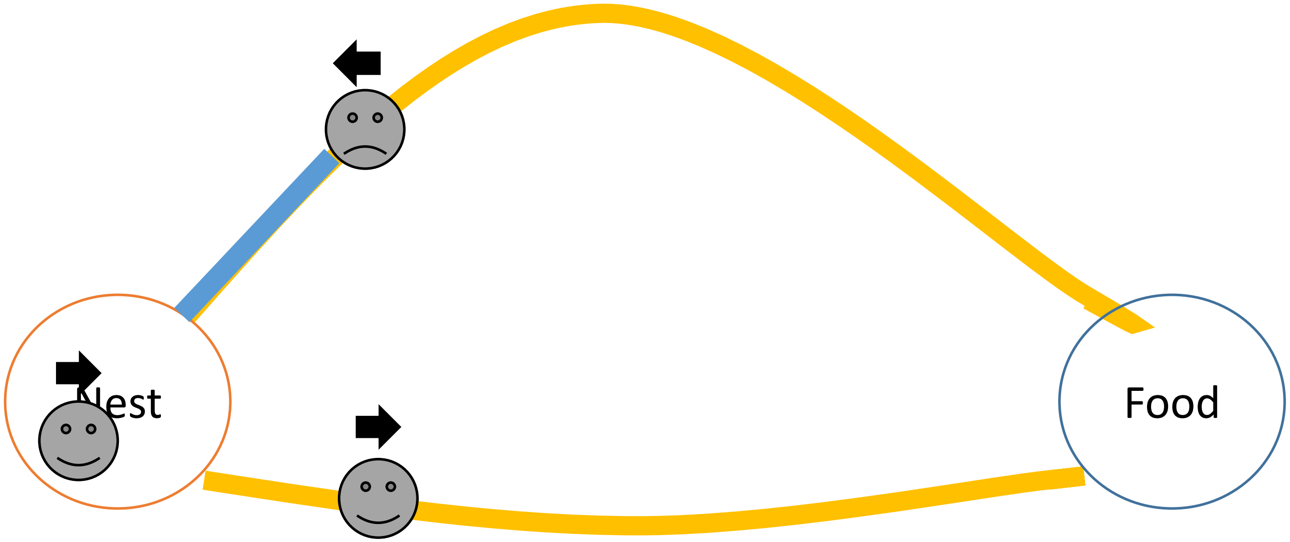


When the ants in the shorter path find a food source, they carry the food and start returning back, following their pheromone trails, and still depositing more pheromone.

Introduction

Ant foraging behavior

- Ant foraging search by pheromone trails

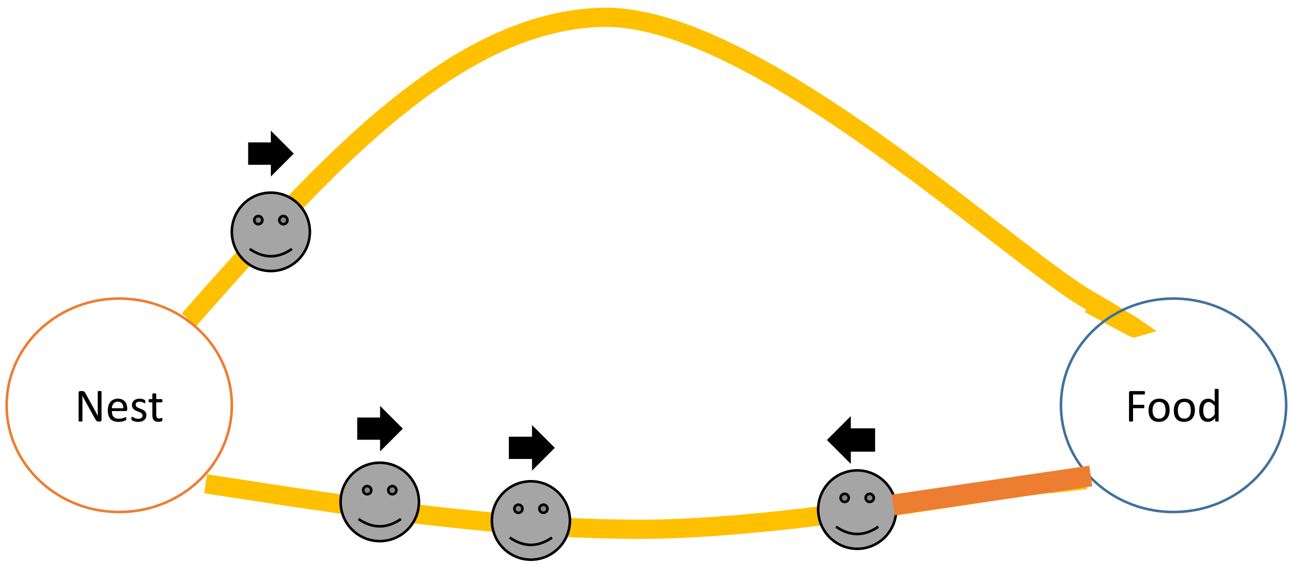


An ant will most likely select the shortest path when returning back to the nest with food as this path will have the most deposited pheromone.

Introduction

Ant foraging behavior

- Ant foraging search by pheromone trails

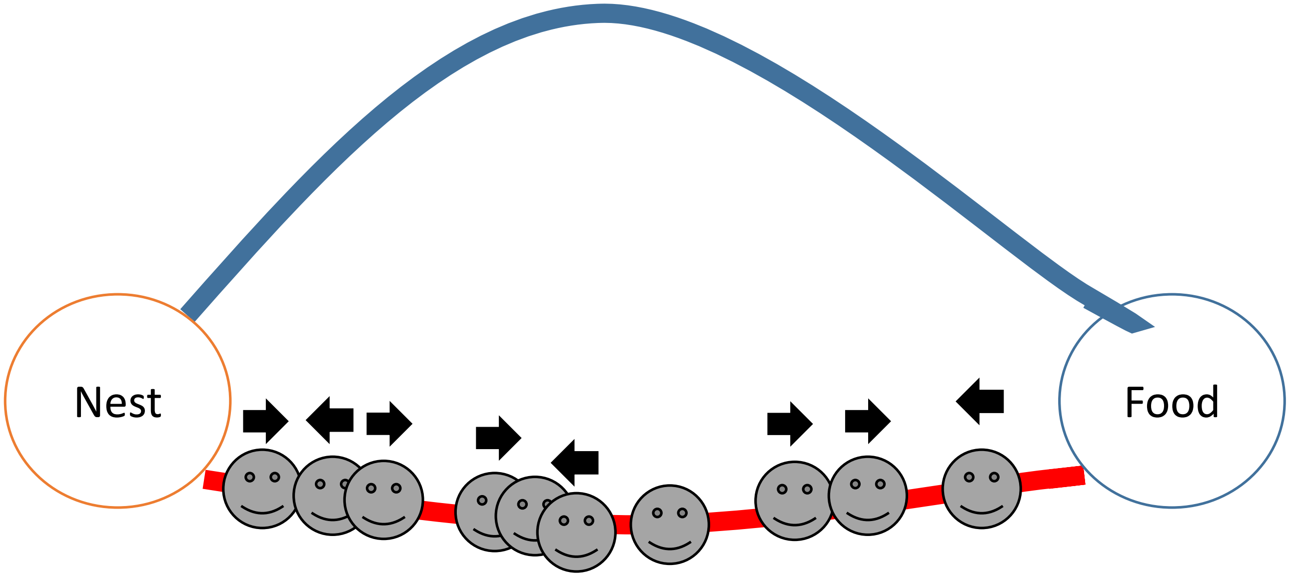


For the same reason, new ants that later starts out from the nest to find food will also select the shortest path.

Introduction

Ant foraging behavior

- Ant foraging search by pheromone trails



Over time, this positive feedback (autocatalytic) process prompts all ants to select the shorter path

Introduction

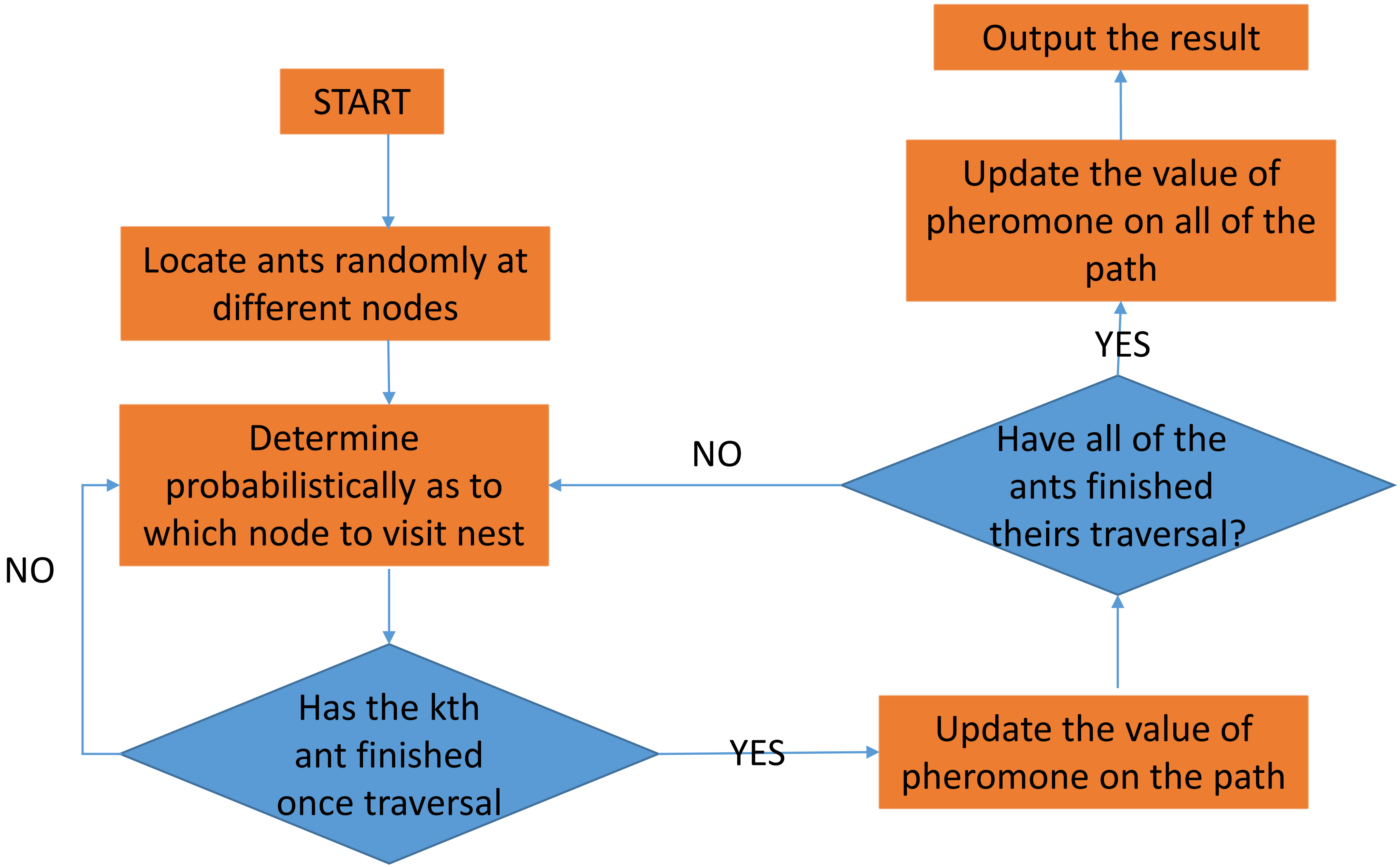
Summary

- Ants will select to follow trails with higher pheromone levels with a higher probability than trails with lower pheromone levels.
- As more ants use a path, the pheromone trail grows stronger.
- This results in autocatalytic behavior, as more ants follow the trail, the trail becomes increasingly attractive.
- This is the main source of inspiration for ACO

ACO Algorithm

- Construct solutions
  - -Explore the search space
  - -Select next path probabilistically according to the pheromone trail
- Apply local search to constructed solutions (Optional)
- Update pheromones
  - -Evaporate about whole pheromone trail
  - -Lay new pheromone on selected path

ACO Algorithm



ACO Algorithm

Construct Solution

- EDGE SELECTION

$$p_{ij}^{k} = \begin{cases} \dfrac{\tau_{ij}^{\alpha} \cdot \mu_{ij}^{\beta}}{\sum_{c_{il} \in N(s^p)} \tau_{ij}^{\alpha} \cdot \mu_{ij}^{\beta}} & if\ c_{ij} \in N(s^p) \\ 0 & otherwise, \end{cases},$$

p_{ij}^{k} : Probability of combination(i,j)
l : a city not yet visited by the ant k.
s^p : the feasible components.
$N(s^p)$: the set of feasible components.
τ_{ij}: Pheromone trail of combination(I,j)
$\mu_{ij} = 1/d_{ij}$: 1/distance of path(i,j)
α, β : control the relative importance of the pheromone versus the heuristic information.

Update pheromone

- PHEROMONE UPDATE

The pheromone τ_{ij} , associated with the edge joining cities I and j, is updated as follows:

$$\tau_{ij} \leftarrow (1-p) \cdot \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k}$$

m : the number of ants

p : the evaporation rate p

$\Delta\tau_{ij}^{k}$: the quantity of pheromone laid on edge (i,j) by ant k

ACO Algorithm

Update pheromone

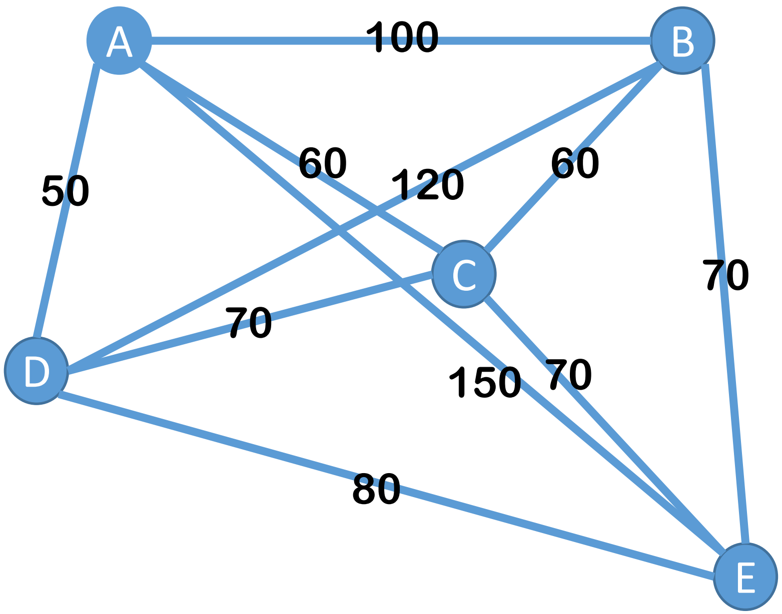
$$p_{ij}^{k} = \begin{cases} \frac{Q}{L_k} & if\ ant\ k\ used\ edge\ (\ i,j) in\ its\ tour, \\ 0 & otherwise, \end{cases},$$

Q : simply parameter to adjust the amount of pheromone deposited

L_k : the cost of the kth ant's tour

TSP using ACO

The goal is to find the shortest round-trip to link a series of cities.
At each stage, the ant chooses to move from one city to another according to some rules

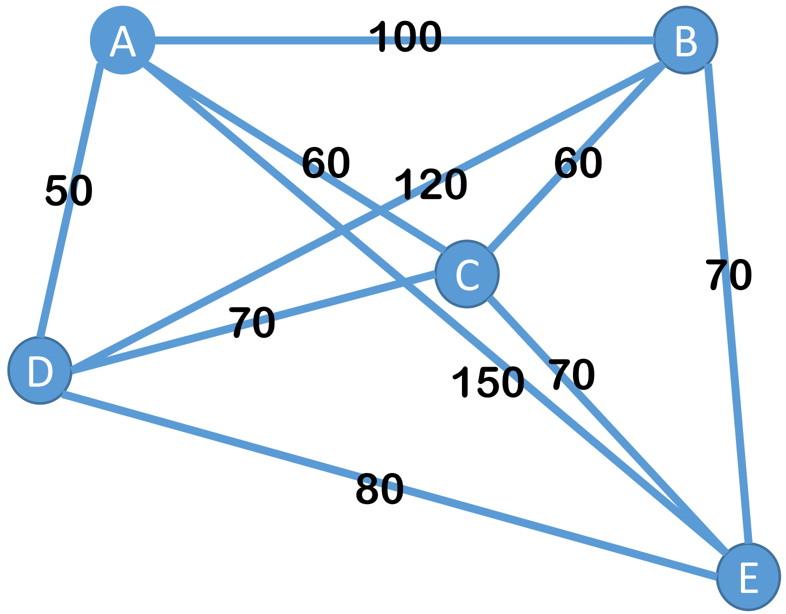


Distances

d_{AB}=100
d_{AC}=60
d_{AD} =50
d_{AE}=150
d_{BC} =60
d_{BD}=120
d_{BE}=70
d_{CD}=70
d_{CE}=70
d_{DE}=80

RULE
-It must visit each city exactly once
-The more intense the pheromone trail laid out on between each edge, the greater the probability that edge will be chosen
-Having completed its journey, the ant deposits more Pheromones on all edges it traversed, if the journey is short
-After each iteration, trails of pheromones evaporate.

TSP using ACO



Distances

d_{AB}=100
d_{AC}=60
d_{AD} =50
d_{AE}=150
d_{BC} =60
d_{BD}=120
d_{BE}=70
d_{CD}=70
d_{CE}=70
d_{DE}=80

Given m= number of ants; n= number of cities
The amount of pheromone on whole paths have same initial value 0.2 .
When first iteration, k number of ants are assigned to each different city.

TSP using ACO

Pheromone Initialization in Iteration 1

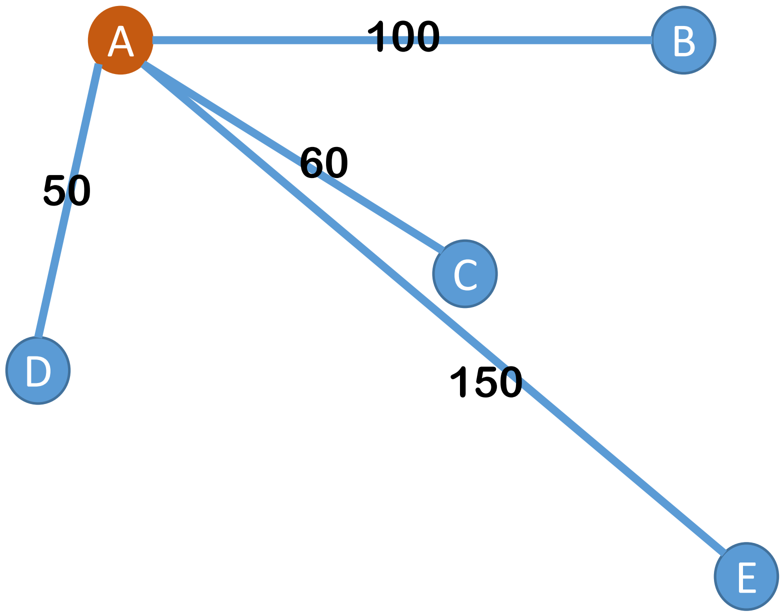
$$\text{PHEROMONE UPDATE} : \tau_{ij} \leftarrow (1-p) \cdot \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k}$$

Assume that p=0.6

Path	Decay Rate
AB	0.2
AC	0.2
AD	0.2
AE	0.2
BC	0.2
BD	0.2
BE	0.2
CD	0.2
CE	0.2
DE	0.2

TSP using ACO

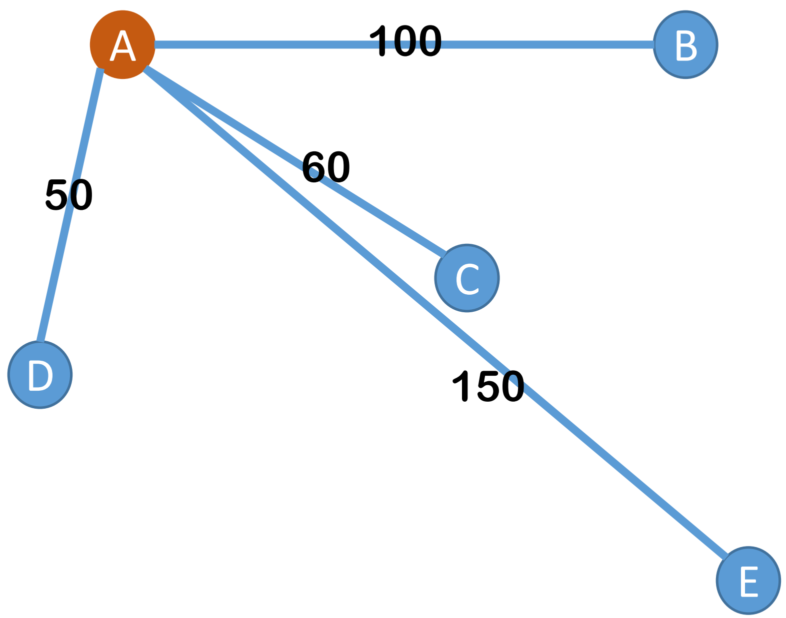
Iteration 1



When first iteration, n number of ants are assigned to each different city.
Ant(1) has four different option.
Ant(1) calculates probabilistic value for the four options by EDGE SELECTION
Ant(1) select path by a highest probabilistic value

TSP using ACO

Iteration 1



EDEGE SELECTION

$$p_{ij}^{k} = \frac{\tau_{ij}^{\alpha} \cdot \mu_{ij}^{\beta}}{\sum_{c_{il} \in N(s^p)} \tau_{ij}^{\alpha} \cdot \mu_{ij}^{\beta}}$$

τ_{ij}^{α}=Amount of Pheromone

μ_{ij}^{β}=1/distance

α=0.2

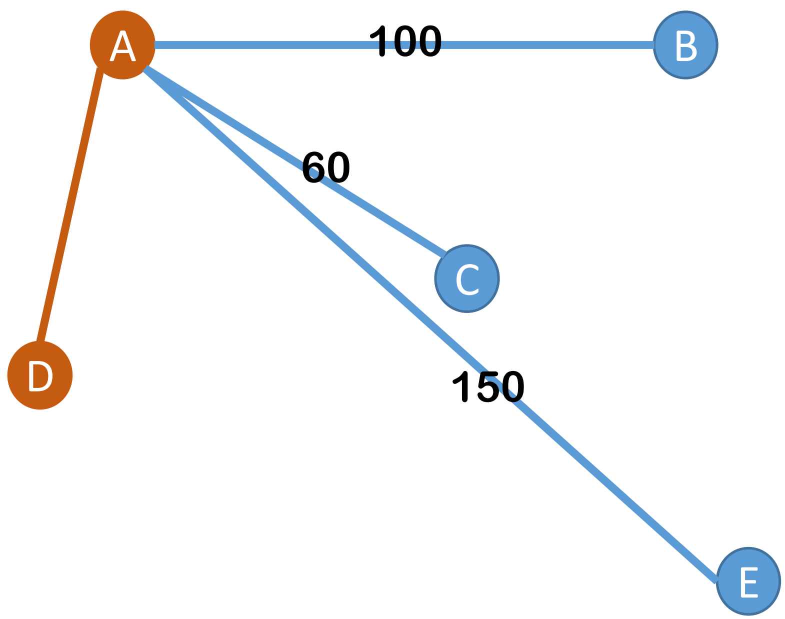
β=0.6

$$p_{AD}^{1} = \frac{0.2^{0.2} \cdot 1/50^{0.6}}{0.2^{0.2} \cdot \frac{1}{100}^{0.6} + 0.2^{0.2} \cdot \frac{1}{60}^{0.6} + 0.2^{0.2} \cdot \frac{1}{150}^{0.6} + 0.2^{0.2} \cdot 1/50^{0.6}}$$

$= 0.2719$, city D has the highest probability.

TSP using ACO

Iteration 1



Ant(1) moved to city D
The pheromone of the whole path evaporates and the pheromone of Path AD increases.

TSP using ACO

Pheromone update in Iteration 1

Assume that p=0.6 , Q= 100

PHEROMONE UPDATE : $\tau_{ij} \leftarrow (1-p)\cdot\tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k}$

$\Delta\tau_{ij}^{k} = \frac{Q}{L_k}$ $\quad if\ ant\ k\ used\ edge\ (\,i, j)in\ its\ tour$

Q= 100
L_1= 50

SO,
$\Delta\tau_{AD}^{1} = 2$

The pheromone on path DC after iteration2 will be updated.

TSP using ACO

Evaporation update in Iteration 1

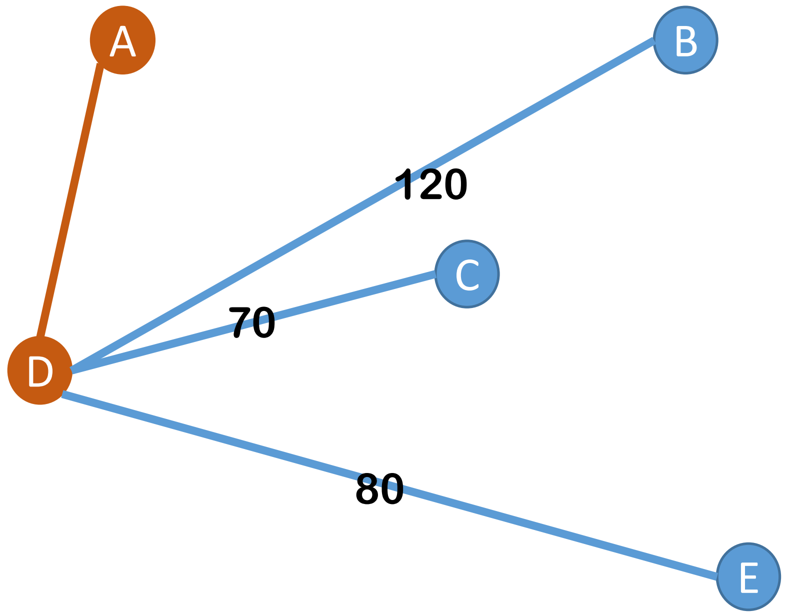
PHEROMONE UPDATE : $\tau_{ij} \leftarrow (1-p) \cdot \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k}$

Assume that p=0.6

Path	Decay Rate
AB	0.08
AC	0.08
AD	2.08
AE	0.08
BC	0.08
BD	0.08
BE	0.08
CD	0.08
CE	0.08
DE	0.08

TSP using ACO

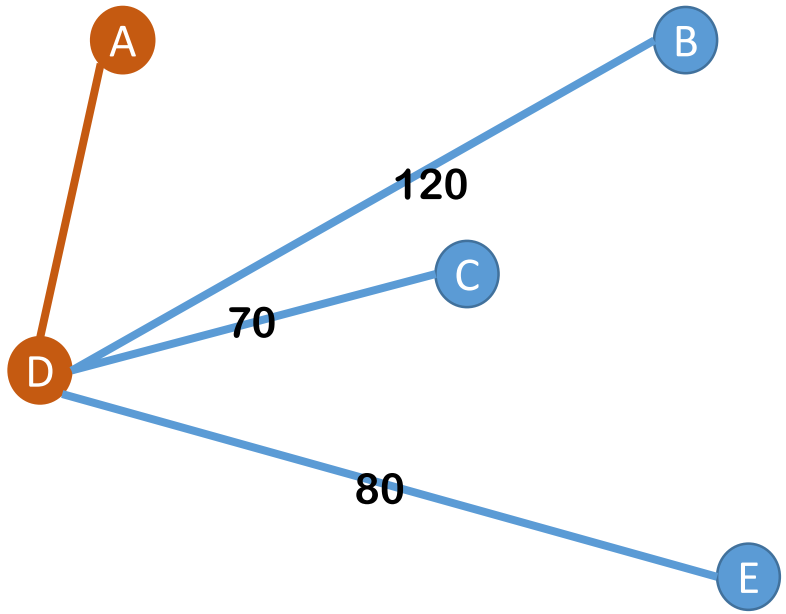
Iteration 2



Now Ant(1) has tree different option.
Again, Ant(1) will do EDGE SELECTION and select path by a highest probabilistic value.
After EDGE SELECTION, the PHEROMONE UPDATE is executed again.

TSP using ACO

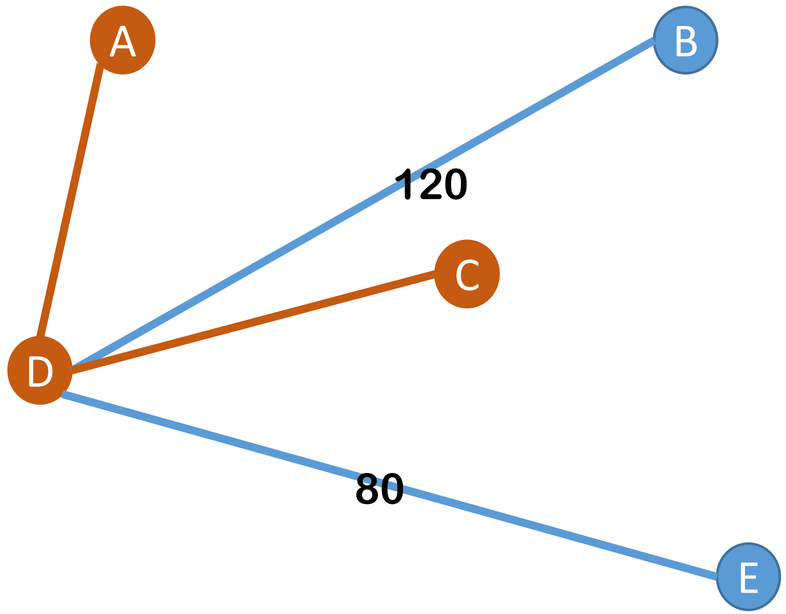
Iteration 2



$p_{CD}^{1} = 0.2528$, city C has the highest probability.

TSP using ACO

Iteration 2



Ant(1) moved to city C
The pheromone of the whole path evaporates and the pheromone of Path AD increases.

TSP using ACO

Pheromone update in Iteration 1

Assume that p=0.6

PHEROMONE UPDATE : $\tau_{ij} \leftarrow (1-p)\cdot\tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k}$

$$\Delta\tau_{ij}^{k} = \frac{Q}{L_k} \qquad if\ ant\ k\ used\ edge\ (\,i,j)in\ its\ tour$$

Q= 100

L_1= 50+70

SO,

$\Delta\tau_{DC}^{1} = 0.7142$

The pheromone on path DC after iteration2 will be updated.

TSP using ACO

Evaporation update in Iteration 2

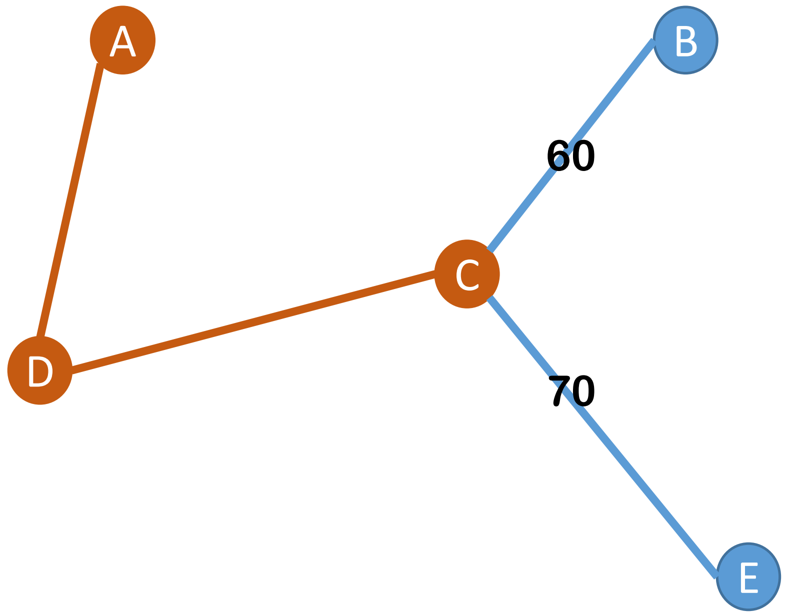
Assume that p=0.6

$$\text{PHEROMONE UPDATE} : \tau_{ij} \leftarrow (1-p) \cdot \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k}$$

Path	Decay Rate
AB	0.032
AC	0.032
AD	0.832
AE	0.032
BC	0.032
BD	0.032
BE	0.032
CD	0.032
CE	0.7462
DE	0.032

TSP using ACO

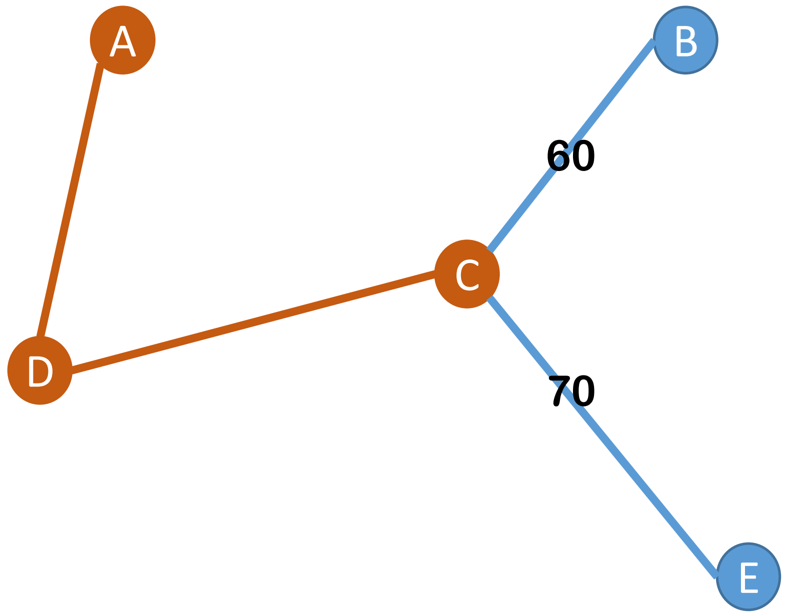
Iteration 3



Now Ant(1) has two different option.
Again, Ant(1) will do EDGE SELECTION and select path by a highest probabilistic value.
After EDGE SELECTION, the PHEROMONE UPDATE is executed again.

TSP using ACO

Iteration 3



The pheromone on path DC
After iteration2 will be
updated by.

$p_{BC}^{1} = 0.5613$, city B has the highest probability.

TSP using ACO

Evaporation update in Iteration 3

Assume that p=0.6

PHEROMONE UPDATE : $\tau_{ij} \leftarrow (1-p) \cdot \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k}$

$\Delta\tau_{ij}^{k} = \frac{Q}{L_k}$ $\quad$ $if\ ant\ k\ used\ edge\ (\ i, j) in\ its\ tour$

SO,

$\Delta\tau_{BC}^{1} = 0.5555$ $\quad$ L_1= 180

The pheromone on path DC after iteration2 will be updated.

TSP using ACO

Evaporation update in Iteration 3

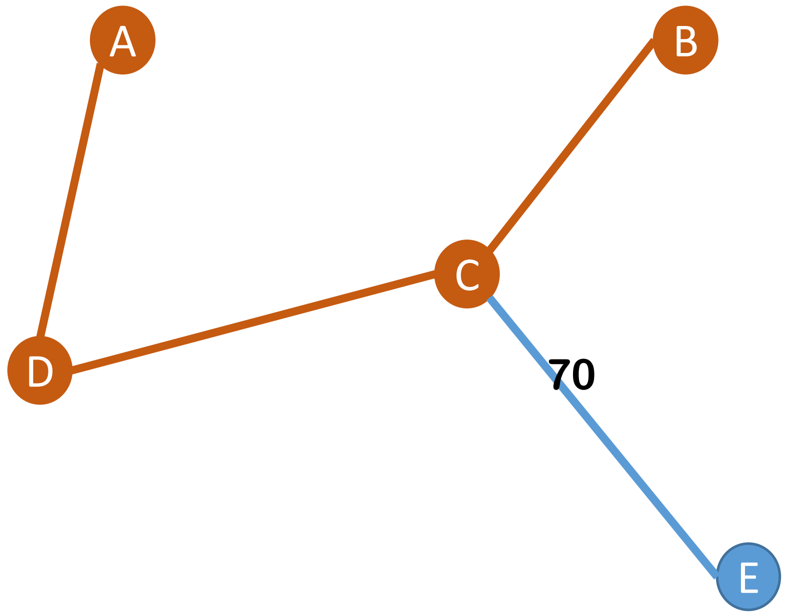
PHEROMONE UPDATE : $\tau_{ij} \leftarrow (1-p) \cdot \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k}$

Assume that p=0.6

Path	Decay Rate
AB	0.0128
AC	0.0128
AD	0.3328
AE	0.0128
BC	0.5683
BD	0.0128
BE	0.0128
CD	0.2984
CE	0.0128
DE	0.0128

TSP using ACO

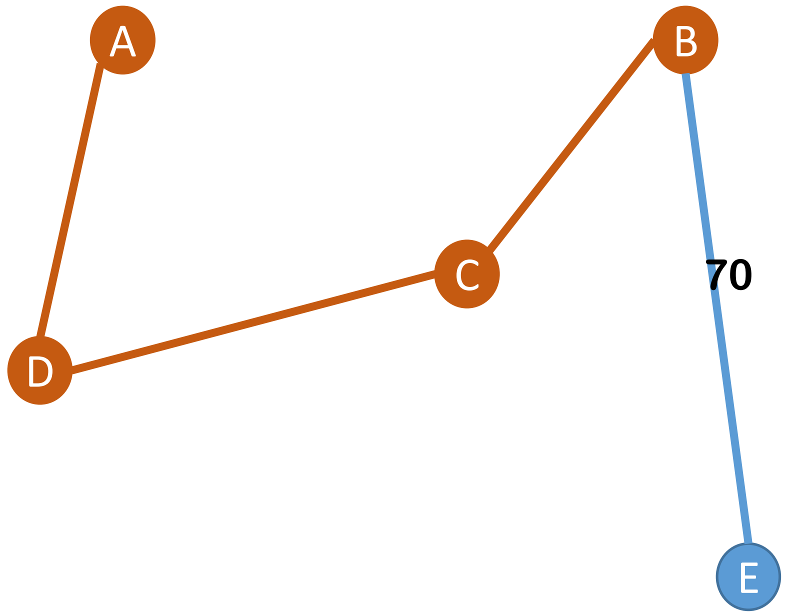
Iteration 3



Ant(1) moved to city B
The pheromone of the whole path evaporates and the pheromone of Path AD increases.

TSP using ACO

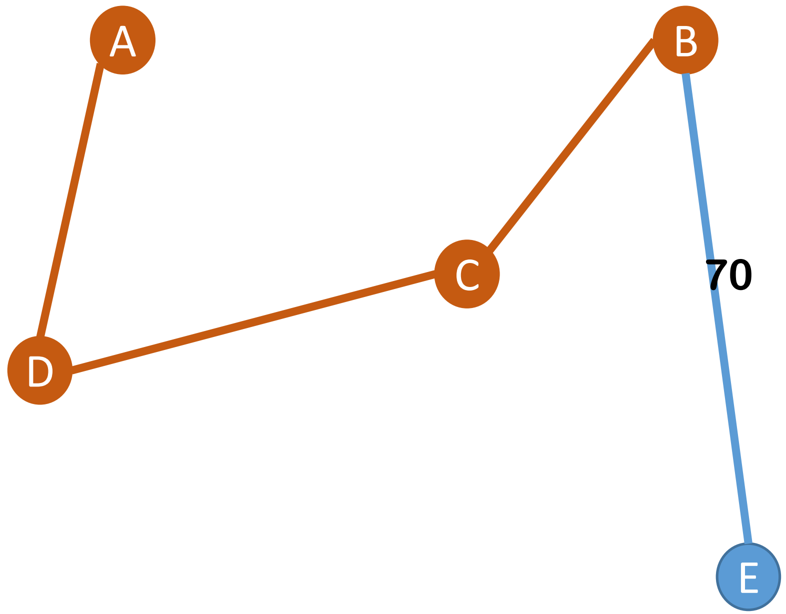
Iteration 4



Now Ant(1) has only one option.
Ant(1) has already visited city A,B,C and D.
Again, Ant(1) will do EDGE SELECTION and select path by a highest probabilistic value.
After EDGE SELECTION, the PHEROMONE UPDATE is executed again.

TSP using ACO

Iteration 4

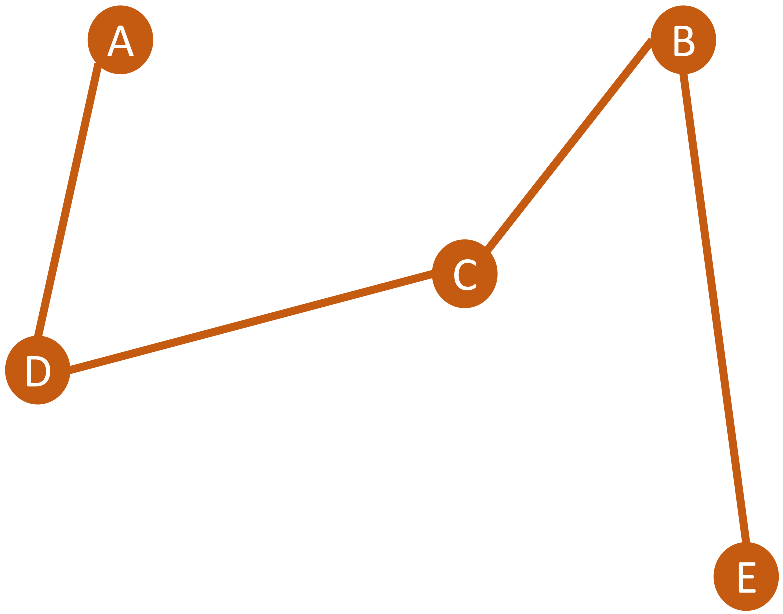


$$p_{BE}^{1} = \frac{0.2^{0.2} \cdot 1/60^{0.6}}{0.2^{0.2} \cdot 1/60^{0.6}}$$

=1, city E has the highest probability.

TSP using ACO

Iteration 4



Ant(1) moved to city E
The pheromone of the whole path evaporates and the pheromone of Path AD increases.

TSP using ACO

Evaporation update in Iteration 4

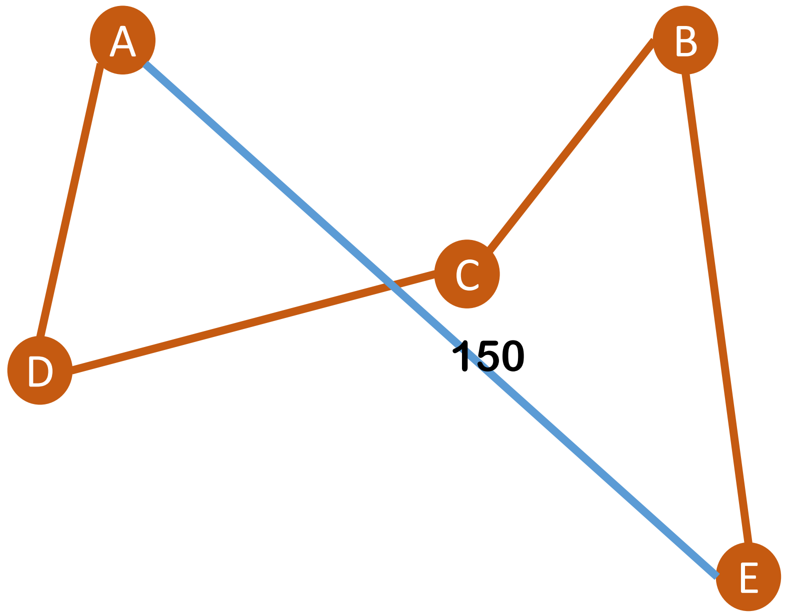
PHEROMONE UPDATE : $\tau_{ij} \leftarrow (1-p) \cdot \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k}$

Assume that p=0.6

Path	Decay Rate
AB	0.00512
AC	0.00512
AD	0.13312
AE	0.00512
BC	0.22732
BD	0.00512
BE	0.42172
CD	0.11936
CE	0.00512
DE	0.00512

TSP using ACO

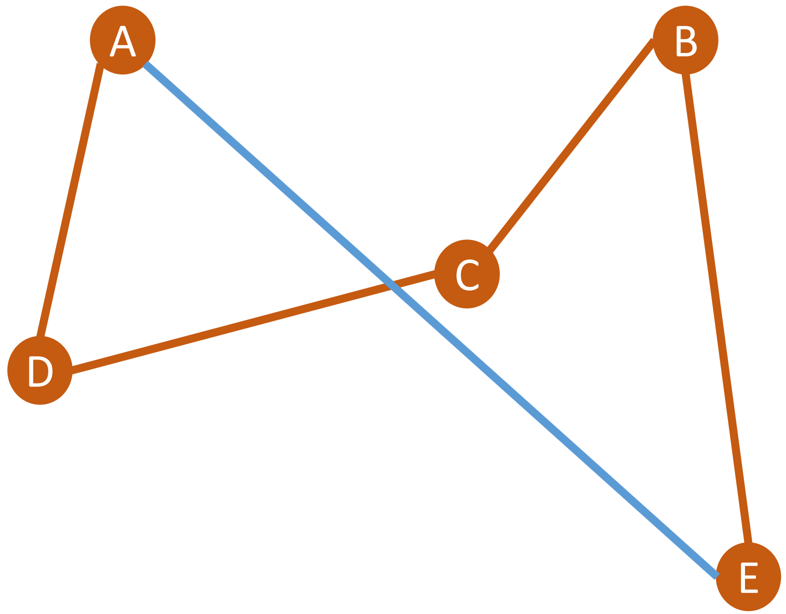
Iteration 5



Now Ant(1) has only one option.
Ant(1) has already visited city A,B,C,D and E.
Again, Ant(1) will do EDGE SELECTION and select path by a highest probabilistic value.
After EDGE SELECTION, the PHEROMONE UPDATE is executed again.

TSP using ACO

Iteration 5



$$p_{AE}^{1} = \frac{0.2^{0.2} \cdot 1/150^{0.6}}{0.2^{0.2} \cdot 1/150^{0.6}}$$

=1, city A has the highest probability.

TSP using ACO

Evaporation update in Iteration 5

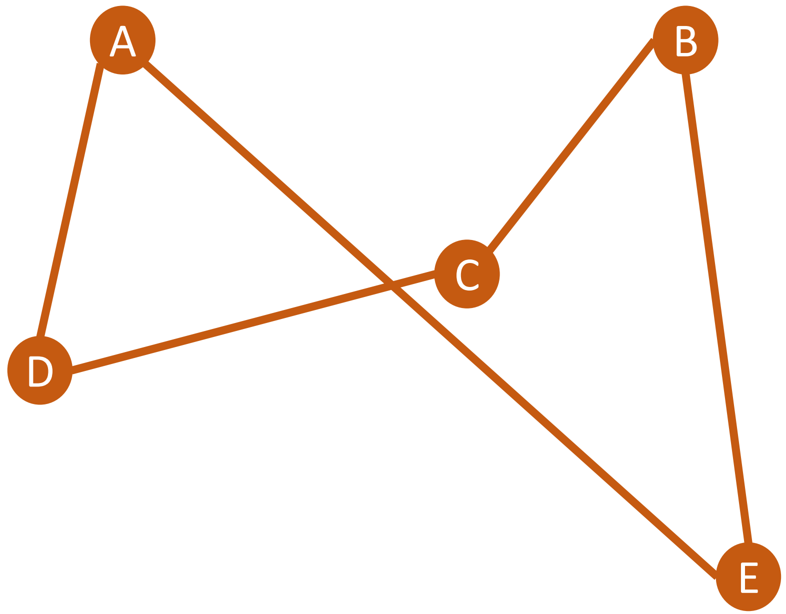
$$\text{PHEROMONE UPDATE}: \tau_{ij} \leftarrow (1-p)\cdot\tau_{ij} + \sum_{k=1}^{m}\Delta\tau_{ij}^{k}$$

Assume that p=0.6

Path	Decay Rate
AB	0.002048
AC	0.002048
AD	0.053248
AE	0.002048
BC	0.090928
BD	0.002048
BE	0.16868
CD	0.047744
CE	0.002048
DE	0.002048

TSP using ACO

Iteration 5



Ant(1) moved to city A
Indicated path is the optimal path by Ant(1).

TSP using ACO

Path by Each Ant

Ant(1) [A,D,C,B,E,A] L= 50+70+60+70+150 = 400

Ant(2) [B,C,A,D,E,B] L= 60+60+50+80+70 = 320

Ant(3) [C,A,D,E,B,C] L= 60+50+80+70+60 = 320

Ant(4) [D,E,A,B,C,D] L= 80+150+100+60+70 = 460

Ant(5) [E,A,B,C,D,E] L= 150+100+60+70+80 = 460

Distances

d_{AB}=100
d_{AC}=60
d_{AD} =50
d_{AE}=150
d_{BC} =60
d_{BD}=120
d_{BE}=70
d_{CD}=70
d_{CE}=70
d_{DE}=80

Here with optimal path length.
Ant(2), Ant(3) found an optimal path in First Run.

Algorithm in Pseudocode

-Initialize Trail
-do while (Stopping Criteria Not Satisfied)
ConstructAndSolutions
ApplyLocalSearch (optional)
UpdatePheromones
end while

- ConstructAntSolutions : Partial solution extended by adding an edge based on stochastic and pheromone considerations.

- ApplyLocalSearch : problem specific, used in state of ACO algorithms.

- UpdatePheromones : increase pheromone of good solutions, decrease that of bad solutions (pheromone evaporation).