A Feature Selection Study for Medical Image deep learning

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Outline

- Deep Feature Selection study
- Thyroid Associated Ophthalmopathy Research
- Industry-University Collaboration Project

Deep Feature Selection study

$$[X\tilde{X}]^T[X\tilde{X}] = \begin{vmatrix} \Sigma & \Sigma - S \\ \Sigma - S & \Sigma \end{vmatrix} \leftrightarrow \tilde{X} = X(\mathrm{I} - \Sigma^{-1}\mathrm{S}) + \widetilde{U}C$$

where $\widetilde{U}$ is orthonormal to the column space of X, $C \in R^{(n-p)\times p}$ satisfies $\mathrm{C}^{\mathrm{T}}C = 2S - S\,\Sigma^{-1}S \geq 0$, S is $\mathrm{diag}\{s\}$ and s=$(s_1, \ldots\, s_p)$ has nonnegative entries.

- Why use a correlation structure

Deep Feature Selection study

$$[X\tilde{X}]^T[X\tilde{X}] = \begin{vmatrix} \Sigma & \Sigma - S \\ \Sigma - S & \Sigma \end{vmatrix} \leftrightarrow \tilde{X} = X(\mathrm{I} - \Sigma^{-1}\mathrm{S}) + \tilde{U}C$$

where $\tilde{U}$ is orthonormal to the column space of X, $C \in R^{(n-p)\times p}$ satisfies $\mathrm{C}^{\mathrm{T}}C = 2S - S\,\Sigma^{-1}S \geq 0$, S is diag$\{s\}$ and s=($s_1, \ldots\ s_p$) has nonnegative entries.

- $X = \begin{vmatrix} 1 \\ 0 \\ -1 \end{vmatrix}, \Sigma = |1|, \Sigma^{-1} = |1|$

- $\tilde{X} = X(\mathrm{I} - \Sigma^{-1}\mathrm{S}) + \tilde{U}C = \begin{vmatrix} 1 \\ 0 \\ -1 \end{vmatrix}(|1| - |1|\mathrm{S}) + \tilde{U}C$

Deep Feature Selection study

- $G = \begin{vmatrix} \Sigma & \Sigma - S \\ \Sigma - S & \Sigma \end{vmatrix}$ is positive semi-definite matrix,

  IFF, $2\Sigma - S \geq 0$ and $2S - S\Sigma^{-1}S \geq 0$

Equi-correlated knockoffs:

$s_j = 2\lambda_{min}(\Sigma) \wedge 1$, where $\tilde{X}_j^T X_j = 1 - s_j, \mathrm{diag}\{s\} \leq 2X^T X$

=> minimize $|(\tilde{X}_j^T \quad X_j)|$

$$s_1 = 2\lambda_{min}(\Sigma) \wedge 1 = 2 \wedge 1 = 1$$

$$S = |1|$$

Deep Feature Selection study

$$[X\tilde{X}]^T[X\tilde{X}] = \begin{vmatrix} \Sigma & \Sigma - S \\ \Sigma - S & \Sigma \end{vmatrix} \quad \leftrightarrow \quad \tilde{X} = X(\mathrm{I} - \Sigma^{-1}\mathrm{S}) + \tilde{U}C$$

where $\tilde{U}$ is orthonormal to the column space of X, $C \in R^{(n-p)\times p}$ satisfies $\mathrm{C}^{\mathrm{T}}C = 2S - S\,\Sigma^{-1}S \geq 0$, S is $\mathrm{diag}\{s\}$ and s=$(s_1, \ldots\, s_p)$ has nonnegative entries.

- $U^T X = 0$ and $U^T U = I$

  - Deterministic U

    1: Perform QR decomposition $\left[X\ 0_{n\times p}\right] = [Q_x\ Q_0]\,R$

    2: $U \leftarrow Q_0$

Deep Feature Selection study

$\widetilde{U}$ is orthonormal to the column space of X, $U^T X = 0$ and $U^T U = I$

1: Perform QR decomposition $\left[X\ 0_{n\times p}\right] = [Q_x\ Q_0]\ R$

$$\left[X\ 0_{n\times p}\right] = \begin{vmatrix} 1 & 0 \\ 0 & 0 \\ -1 & 0 \end{vmatrix}$$

- QR decomposition:

  A=QR

  where A is a $m \times n$ complex matrix, Q is a $m \times n$ unitary matrix and R is a $\mathrm{n} \times n$ upper triangular matrix.

  If A is real matrix, then, Q is orthogonal matrix.

  If A has n linearly independent columns, then the first n columns of Q form an orthonormal basis for the column space of A.

Deep Feature Selection study

- QR decomposition: A=QR

  If A has n linearly independent columns, then the first n columns of Q form an orthonormal basis for the column space of A.

- Gram-Schmidt process:

  Given linearly independent set of vectors $\{v_1, \dots, v_k\}$ in an inner product space V (i.e. $\mathbb{R}^n$), calculate orthonormal basis $\{e_1, \dots, e_k\}$

$$u_1 = v_1$$
$$u_2 = v_2 - proj_{u_1}(v_2)$$
$$u_3 = v_3 - proj_{u_1}(v_3) - proj_{u_2}(v_3)$$
$$\vdots$$
$$u_k = v_k - \sum_{j=1}^{k-1} proj_{u_j}(v_k)\,, e_k = \frac{u_k}{||u_k||}$$

Deep Feature Selection study

$\widetilde{U}$ is orthonormal to the column space of X, $U^T X = 0$ and $U^T U = I$

1: Perform QR decomposition $\left[X\ 0_{n\times p}\right] = [Q_x\ Q_0]\ R$

$$[Q_x^T Q_0^T]\left[X\ 0_{n\times p}\right] = R$$

$$\begin{bmatrix} Q_x^T\ X & Q_x^T\ 0 \\ Q_0^T\ X & Q_0^T\ 0 \end{bmatrix} = \begin{bmatrix} Q_x^T\ X & 0 \\ Q_0^T\ X & 0 \end{bmatrix} = R$$

$\because$ If $A = QR$ is QR decomposition of A, $Q^T Q = I$ and R is an upper triangular matrix

2: $U \leftarrow Q_0$

Deep Feature Selection study

$\widetilde{U}$ is orthonormal to the column space of X, $U^T X = 0$ and $U^T U = I$

1: Perform QR decomposition $\left[X\ 0_{n\times p}\right] = [Q_x\ Q_0]\ R$

$$\left[X\ 0_{n\times p}\right] = \begin{vmatrix} 1 & 0 \\ 0 & 0 \\ -1 & 0 \end{vmatrix} = \begin{vmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{vmatrix} R$$

- Thin decomposition phase:

$$u_1 = v_1 = (1, 0, -1)\ \therefore q_{11} = \frac{1}{\sqrt{2}}\ q_{21} = 0,\ q_{31} = -\frac{1}{\sqrt{2}}$$

$$u_2 = v_2 - proj_{u_1}(v_2) = (0, 0, 0)\ \therefore q_{12} = 0\ q_{22} = 0,\ q_{32} = 0$$

- Full decomposition phase:

$$q_{11} \times q_{13} + q_{21} \times q_{23} + q_{31} \times q_{33} = 0$$

$$q_{12} \times q_{13} + q_{22} \times q_{23} + q_{32} \times q_{33} = 0$$

Deep Feature Selection study

$\widetilde{U}$ is orthonormal to the column space of X, $U^T X = 0$ and $U^T U = I$

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- Full decomposition phase:

$$\frac{1}{\sqrt{2}} \times q_{13} + 0 \times q_{23} - \frac{1}{\sqrt{2}} \times q_{33} = 0$$

$$0 \times q_{13} + 0 \times q_{23} + 0 \times q_{33} = 0 \quad \therefore q_{13} = q_{33}$$

$$q_{13} = q_{33} = t, \sqrt{t^2 + t^2 + q_{23}^2} = 1$$

$$=> t = \frac{1}{\sqrt{2}}, \therefore q_{13} = \frac{1}{\sqrt{2}}, q_{23} = 0, q_{33} = \frac{1}{\sqrt{2}}$$

Deep Feature Selection study

$\tilde{U}$ is orthonormal to the column space of X, $U^T X = 0$ and $U^T U = I$

1: Perform QR decomposition $\left[X\ 0_{n\times p}\right] = [Q_x\ Q_0]\ R$

$$\left[X\ 0_{n\times p}\right] = \begin{vmatrix} 1 & 0 \\ 0 & 0 \\ -1 & 0 \end{vmatrix} = \begin{vmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{vmatrix} R$$

- Reassign q_{12} , q_{22} and q_{32} :

$$\frac{1}{\sqrt{2}} \times q_{12} + 0 \times q_{22} - \frac{1}{\sqrt{2}} \times q_{32} = 0 \text{ -> } q_{12} - q_{32} = 0$$

$$\frac{1}{\sqrt{2}} \times q_{12} + 0 \times q_{22} + \frac{1}{\sqrt{2}} \times q_{32} = 0 \text{ -> } q_{12} + q_{32} = 0$$

$\therefore\ q_{12} = q_{32} = 0,\ q_{22} = 1$ for $U^T U = I$

Deep Feature Selection study

$\widetilde{U}$ is orthonormal to the column space of X, $U^T X = 0$ and $U^T U = I$

1: Perform QR decomposition $\left[X\ 0_{n\times p}\right] = [Q_x\ Q_0]\ R$

$$\left[X\ 0_{n\times p}\right] = \begin{vmatrix} 1 & 0 \\ 0 & 0 \\ -1 & 0 \end{vmatrix} = \begin{vmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{vmatrix} R$$

2: $U \leftarrow Q_0$

Q_0 can be $(0,1,0)$ or $(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}})$ by Gram–Schmidt process.

U= $(0,1,0)$, $(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}})$

Deep Feature Selection study

$$[X\tilde{X}]^T[X\tilde{X}] = \begin{vmatrix} \Sigma & \Sigma - S \\ \Sigma - S & \Sigma \end{vmatrix} \quad \leftrightarrow \quad \tilde{X} = X(\mathrm{I} - \Sigma^{-1}\mathrm{S}) + \widetilde{U}C$$

where $\widetilde{U}$ is orthonormal to the column space of X, $C \in R^{(n-p)\times p}$ satisfies $\mathrm{C}^{\mathrm{T}}C = 2S - S\,\Sigma^{-1}S \geq 0$, S is $\mathrm{diag}\{s\}$ and s=$(s_1, \ldots\ s_p)$ has nonnegative entries.

Thyroid Associated Ophthalmopathy Research

- 완료 현황: 현재 실개수 카운팅 진행중

	눈	내직근	외직근	시신경
AX 1	약 90%	약 50%	약 50%	약 50%
AX 2		약 90%	약 90%	약 90%
AX 4		약 90%	약 90%	약 90%
AX 5		약 50%	약 50%	약 50%

	4 근육 및 시신경
CO 2	모두 90% 이상
CO 3	전체적으로 실패

- 향후 진행해야할 업무:
  - 자동으로 완료한 슬라이스 검토 작업 및 재처리
  - CO3 세그멘테이션
  - AX 1~5종 슬라이스 지방 세그멘테이션
    - 안와 아웃라인 세그멘테이션 이후 내부구조물 제외 작업
  - 완료된 데이터셋 바탕으로 Activity 파일럿 연구

Industry-University Collaboration Project

- 제출 실적
  - SW 제작 (산학 1), 국제학술대회 논문 1편 이상 게재 (산학 2), 국내 특허출원 1건 이상 (산학 2)
- 현재 진행 사항
  - SW 제작: 추가 실험 진행과 함께 진행할 계획
  - 투고 저널 리스트업
    - 바로 진행 (다음주 금요일)
    - 저널명, 게재 기간(승인 기간), 투고 기간, Impact Factor, 게재료, 투고료, 가입비 등
  - 최대한 기존 논문을 바탕으로, 시간 오래 끌지 않는 방향으로, 논문 작성 완료에 힘써볼 계획

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